Physics of Stochastic Processes

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Inhalt

Physik der Materie — Physik des Straßenverkehrs

Es gibt nur eine Physik, aber die Physik muss zur Fragestellung passen.

1. Makro-Materie: Newtonsche Axiome:
Symmetrie der Wechselwirkung → harmonischer Oszillator → Pendelkette

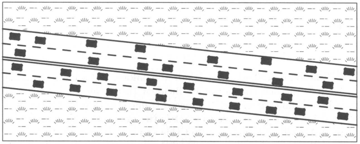
2. Mikro-Materie: Quantenphysik:
Schrödingergleichung → Quantenzahlen inklusive Spin → Termschema

3. Makroskopische Auto-Materie: Straßenverkehrsphysik:
- Starke Asymmetrie der Wechselwirkung (Autos fahren vorwärts) → Nicht-Newton
- Aktive Teilchen (Autos haben einen Tank) → Energie-Bilanz im offenen System
- Verkehrszusammenbruch (empirische Beobachtung) → Stochastischer Prozess

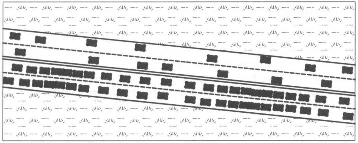
⟹ Langevin-Gleichung — Master-Gleichung — Fokker-Planck-Gleichung

Freier Verkehr

Verkehrszusammenbruch



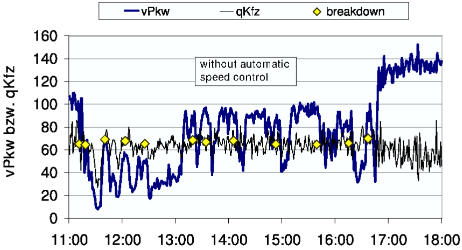
Freier Verkehr bei geringer Dichte



Auto-Stau-Cluster bei hoher Dichte

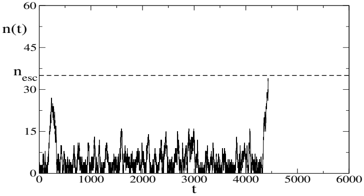
Quelle: Mobility and Traffic in the 21st Century

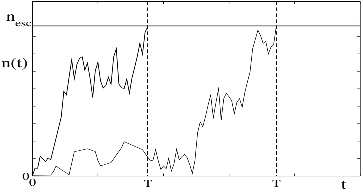
Controlling traffic breakdowns (ISTTT 16, U of Maryland, 2005)



A traffic breakdown is defined (usually based on 5 minutes measurement interval data) as a

- speed drop $\Delta v > 15$ km/h
- mean velocity after speed drop $v_{final} < 75$ km/h
- traffic volume before speed drop $q > 1000$ veh/h/lane

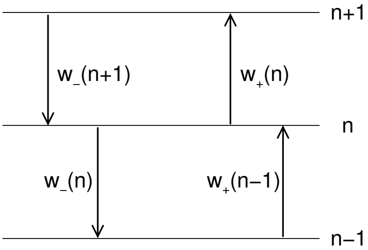




Stochastic Master Equation

Balance equation: probability $P(n,t)$ of number of congested vehicles n at time t

$$\frac{\partial P(n,t)}{\partial t} = w_+(n-1)P(n-1,t) + w_-(n+1)P(n+1,t) - [w_+(n) + w_-(n)]\, P(n,t)$$



Our car cluster model:

- inflow or attachment rate

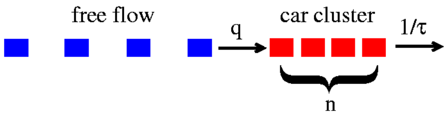
$$w_+(n) = q$$

- escape or detachment rate

$$w_-(n) = 1/\tau$$

Nucleation on Highways

Probabilistic description of traffic pattern formation



q [veh/h] = traffic flow or traffic volume (from net time gap for a freely moving car)

n = cluster size or queue length (number of congested vehicles) as stochastic variable

τ [$\tau \approx 1.5$ s] = characteristic time needed for the first car leaving the cluster to become free

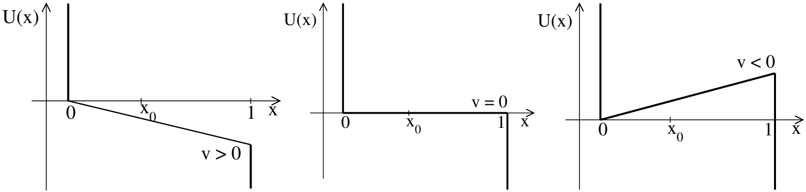
Fokker-Planck-Equation: Drift-Diffusion Approximation

dimensionless quantities: $x = n/n_{esc}$, $T = D\,t$

$$\frac{\partial p(x,T)}{\partial T} = -v\,\frac{\partial p(x,T)}{\partial x} + \frac{\partial^2 p(x,T)}{\partial x^2} \quad \text{with} \quad v = \left(q - \frac{1}{\tau}\right)\frac{1}{D}$$

left boundary: $j(x=0,T) = 0$; right boundary: $p(x=1,T) = 0$

linear potential: $U(x) = -\,v\,x$



$q > 1/\tau$ $q = 1/\tau$ $q < 1/\tau$

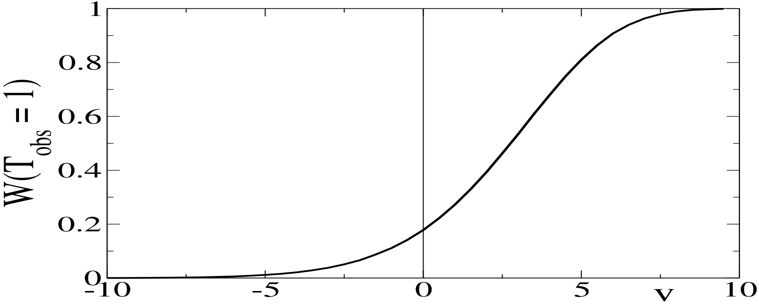
Cumulative breakdown distribution

Defining

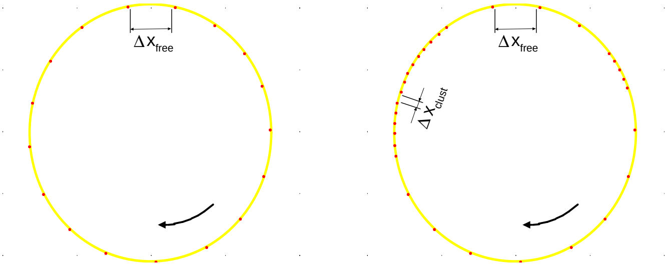
$$W(T_{obs}, v) = \int_0^{T_{obs}} \mathcal{P}(T; v)\, dT$$

in relation to measurements. T_{obs} is called observation time.

See data: Brilon & Zurlinden (2002), Weibull distribution as fit function.



Das einfachste Modell der Auto-Vielteilchen-Physik: N Autos im Kreisverkehr der Länge L



Freier Verkehr (links) und gestauter Stopp-und-Go–Verkehr (rechts) auf einer einspurigen Ringstraße. Jeder rote Punkt repräsentiert ein Auto; die Fahrtrichtung ist durch einen Pfeil angedeutet.

Many–particle chain system with forward– and backward–directed forces

Particle chain dynamics

We consider a many–particle system (particles of mass m numbered by $i = 1, 2, \ldots, N$) with periodic boundary conditions ($0 \leq x < L$). The dynamics is given by the equations of motion for velocities v_i and headway distances Δx_i

$$m \frac{dv_i}{dt} = F_{forw}(\Delta x_i) + F_{back}(\Delta x_{i-1}) + F_{diss}(v_i) \quad (1)$$

$$\frac{d(\Delta x_i)}{dt} = v_{i+1} - v_i \quad ; \quad \Delta x_i = x_{i+1} - x_i \quad ; \quad \sum_{i=1}^{N} \Delta x_i = L \quad (2)$$

with forward directed force F_{forw}, backward directed force F_{back}, as well as dissipation (friction) term $F_{diss}(v_i)$.

Many–particle chain model with forward– and backward–directed forces: Spring–block model

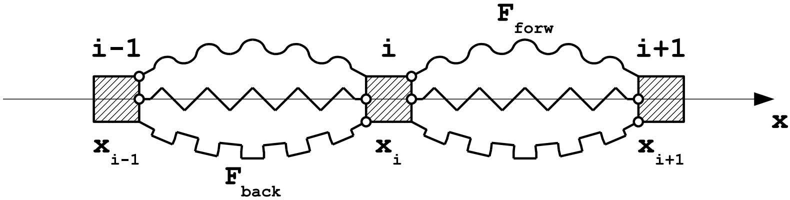
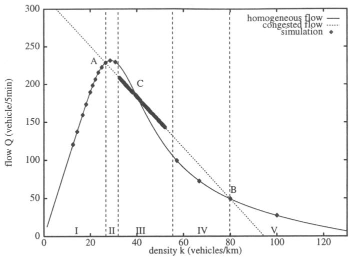


Abb.: A chain model of particles coupled by springs. The forward– and backward–directed forces are represented as different springs marked by F_{forw} and F_{back}, respectively. Each of such springs connects two particles, but acts only on one of them – that particle for which the connection is marked by a circle. The zig–zag spring in the middle shows the usual symmetric interaction.

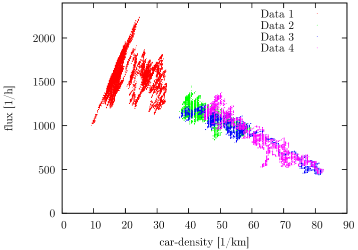
Fundamentaldiagramm

Fluss–Dichte–Relation



Theorie + Simulation

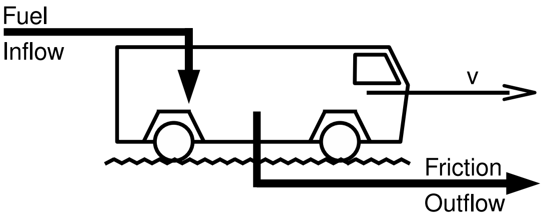
M. Bando et al., J. Phys. (France) 5, 1389 (1995)



Empirische Daten

Christof Liebe, Diplomarbeit, Univ. Rostock (2006)

Auto als offenes System: Energie-Fluss-Schema



Summary

- Traffic as driven many particle system with asymmetric interaction
- Modeling of vehicular transport with active particles
- Level of description as microscopic, mesoscopic or macroscopic system
- Understanding of traffic breakdown as stochastic process